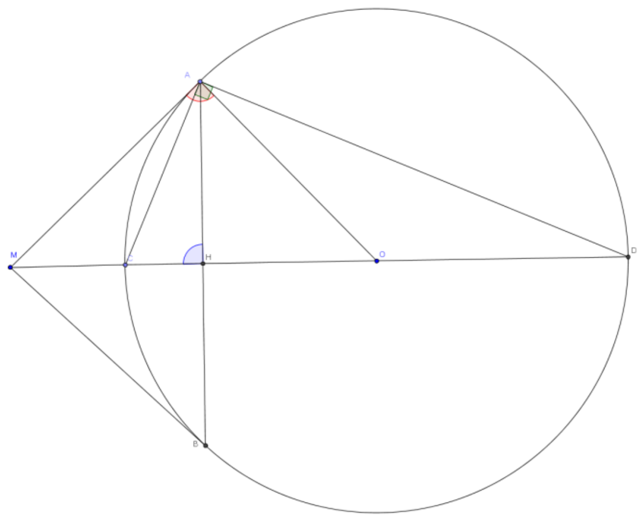

נסמן

$MC = y$

$CH = x$

$HO = R - x$

$OD = R$

נתבקשנו להראות

$$\frac{MC}{MD} = \frac{HC}{HD}$$

$$\frac{y}{2R+y} = \frac{x}{2R-x} \rightarrow 2Ry - xy = 2Rx + xy$$

$$2R(y-x) = 2xy$$

$$R = \frac{xy}{y-x}$$

ע"פ דמיון:

$$MO \cdot MH = AM^2 = MC \cdot MD$$

$$(R+y)(y+x) = y(2R+y)$$

$$Ry + y^2 + Rx + yx = 2Ry + y^2 \rightarrow x(R+y) = Ry$$

$(AH = HB)$

$$CH \cdot HD = AH^2 = MH \cdot HO$$

$$x(2R-x) = (x+y)(R-x)$$

$$2Rx - x^2 = xR + yR - x^2 - xy$$

$$Rx = y(R-x)$$

נחלק את המשוואות זו בזו

$$\frac{Ry}{Rx} = \frac{x(R+y)}{y(R-x)} \rightarrow \frac{y^2}{x^2} = \frac{R+y}{R-x}$$

$$y^2R - y^2x = Rx^2 + x^2y$$

$$xy(x+y) = R(x^2-y^2)$$

$$R = \frac{xy(x+y)}{(x-y)(x+y)} = \frac{xy}{x-y}$$

(S,S) $\triangle AMO \sim \triangle HAO$

$$\frac{OH}{AO} = \frac{AO}{MO} \rightarrow AO^2 = a^2 = OH \cdot MO$$