

(1)

$\triangle ECA \sim \triangle EBC \sim \triangle CBA$ S.S

$$\Rightarrow b^2 = AB \cdot EA \rightarrow AE = \frac{b^2}{c}$$

$$EC^2 = EA \cdot EB = \frac{b^2}{c} (b^2 + c) = \frac{b^4}{c^2} + b^2 = b^2 \left(\frac{b^2}{c^2} + 1 \right)$$

$$EC = b \sqrt{\frac{b^2}{c^2} + 1} = b \sqrt{\frac{b^2 + c^2}{c^2}} = \frac{ba}{c}$$

$\triangle CBD \sim \triangle CAB \sim \triangle BAD$

$$AB^2 = AC \cdot AD \rightarrow AD = \frac{c^2}{b}$$

$$BD^2 = CD \cdot AD = \frac{c^2}{b} \left(\frac{c^2}{b} + b \right) = \frac{c^4}{b^2} + c^2 = c^2 \left(\frac{c^2}{b^2} + 1 \right)$$

$$BD = c \sqrt{\frac{c^2}{b^2} + 1} = c \sqrt{\frac{c^2 + b^2}{b^2}} = \frac{ca}{b}$$

(2)

נתק בטוח לזוהי קשתו בריבוי

$$S_{AEC} = \frac{1}{2} r_1 \left(b + \frac{b^2}{c} + \frac{ba}{c} \right) = \frac{b \cdot b^2}{2c} \rightarrow r_1 = \frac{b^3}{c \left(b + \frac{b^2}{c} + \frac{ba}{c} \right)}$$

$$S_{ABD} = \frac{1}{2} r_2 \left(c + \frac{c^2}{b} + \frac{ca}{b} \right) = \frac{c \cdot c^2}{2b} \rightarrow r_2 = \frac{c^3}{b \left(c + \frac{c^2}{b} + \frac{ca}{b} \right)}$$

$$r_1 + r_2 = \frac{b^3}{cb + b^2 + ba} + \frac{c^3}{bc + c^2 + ca} = \frac{b^3}{b(c + b + a)} + \frac{c^3}{c(b + c + a)} = \frac{b^2 + c^2}{a + b + c} = \frac{a^2}{a + b + c}$$