2.69

$$\frac{3\cdot 2^n}{2!}+\frac{5\cdot 2^{n-1}}{3!}+\dots+\frac{(2n+1)\cdot 2}{(n+1)!}+\frac{(2n+3)\cdot 2^0}{(n+2)!}=2^{n+1}+\frac{(-1)^{2n+1}}{(n+2)!}$$

$n=1$ נוכיח

$$\frac{(2\cdot 1+1)\cdot 2}{2!}+\frac{(2\cdot 1+3)\cdot 1}{(1+2)!}=2^{1+1}+\frac{(-1)^{2\cdot 1+1}}{(1+2)!}$$

$$3+\frac{5}{3!}=4-\frac{1}{3!}$$

$$\frac{18+5}{6}=\frac{24-1}{6}\ \checkmark$$

$n+1$ נניח עבור

$$\frac{3\cdot 2^{n+1}}{2!}+\frac{5\cdot 2^{n+1-1}}{3!}+\dots+\frac{(2(n+1)+1)\cdot 2^1}{(n+2)!}+\frac{[2(n+1)+3]\cdot 2^0}{(n+1+2)!}\overset{?}{=}2^{n+1+1}+\frac{(-1)^{2(n+1)+1}}{(n+1+2)!}$$

$\div 2$

$$\frac{3\cdot 2^{n+1}}{2!\cdot 2}+\frac{5\cdot 2^n}{3!\cdot 2}+\dots+\frac{(2n+3)\cdot 2}{(n+2)!\cdot 2}+\frac{(2n+5)\cdot 2^0}{(n+3)!\cdot 2}\overset{?}{=}\frac{2^{n+2}}{2}+\frac{(-1)^{2n+3}}{(n+3)!\cdot 2}$$

$$\underbrace{\frac{3\cdot 2^n}{2!}+\frac{5\cdot 2^{n-1}}{3!}+\dots+\frac{(2n+3)\cdot 2}{(n+2)!}}+\frac{(2n+5)2^0}{(n+3)!\cdot 2}\overset{?}{=}2^{n+1}+\frac{(-1)^{2n+2}}{(n+3)!}$$

הנחה

$$2^{n+1}+\frac{(-1)^{2n+1}}{(n+2)!}+\frac{(2n+5)2^0}{(n+3)!\cdot 2}\overset{?}{=}2^{n+1}+\frac{(-1)^{2n+2}}{(n+3)!}$$

$$\frac{-2(n+3)+2n+5}{(n+3)!\cdot 2}\overset{?}{=}\frac{1}{(n+3)!}$$

$$\frac{2}{(n+3)!\cdot 2}=\frac{1}{(n+3)!}$$

$$\frac{1}{(n+3)!}=\frac{1}{(n+3)!}$$