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$$n=1 \quad \frac{1}{2} \leq \frac{2}{4} \quad \checkmark$$

$$n=k+1 \quad \frac{(k+1)!^2}{(2k+2)!} \stackrel{?}{\leq} (k+2) \cdot 4^{-k-1}$$

$$\frac{(k+1)^2 (k+1)^2}{(2k)!(2k+1)(2k+2)} \stackrel{?}{\leq} (k+2) 4^{-k-1}$$

$$\frac{(k+2)}{4^{k+1}} \stackrel{?}{\geq} \frac{(k+1) 4^{-k} (k+1)^2}{(2k+1) 2 (k+1)} = \frac{(k+1)^2}{4^k \cdot 2 (2k+1)} \rightarrow \begin{aligned} & 2(k+2)(2k+1)4^k \geq 4^{k+1} (k+1)^2 \\ & 2(2k^2+5k+2) \geq 4(k^2+2k+1) \\ & 4k^2+10k+4 \geq 4k^2+8k+4 \\ & \boxed{2k \geq 0} \end{aligned}$$