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$$n=1 \quad \frac{x}{1-x^2} = \frac{1}{1-x} \cdot \frac{x-x}{1-x^2} \\ = \frac{1}{1-x} \cdot \frac{x(1-x)}{1-x^2}$$

$$n=k+1 \quad \underbrace{\frac{x}{1-x^2} + \dots + \frac{x^{2^{k-1}}}{1-x^{2^k}}}_{\frac{x}{1-x^2} + \dots + \frac{x^{2^{k-1}}}{1-x^{2^k}}} = \frac{x^{2^k}}{1-x^{2^{k+1}}} \stackrel{?}{=} \frac{1}{1-x} \cdot \frac{x-x^{2^{k+1}}}{1-x^{2^{k+1}}}$$

$$\frac{1}{1-x} \cdot \frac{x-x^{2^k}}{1-x^{2^k}} + \frac{x^{2^k}}{1-x^{2^{k+1}}} \stackrel{?}{=} \frac{1}{1-x} \cdot \frac{x-x^{2^{k+1}}}{1-x^{2^{k+1}}}$$

$$\frac{1}{1-x} \cdot \frac{x-x^{2^k}}{1-x^{2^k}} \stackrel{?}{=} \frac{-x^{2^k}(1-x) + x-x^{2^{k+1}}}{(1-x)(1-x^{2^{k+1}})}$$

$$\frac{1}{1-x} \cdot \frac{x-x^{2^k}}{1-x^{2^k}} \stackrel{?}{=} \frac{x-x^{2^k} + x^{1+2^k} - (x^{2^k})^2}{(1-x)(1-x^{2^{k+1}})}$$

$$\stackrel{?}{=} \frac{x(1+x^{2^k}) - x^{2^k}(1+x^{2^k})}{(1-x)(1-x^{2^{k+1}})} = \frac{(x-x^{2^k})(1+x^{2^k})}{(1-x)(1-x^{2^{k+1}})}$$

$$\downarrow = \frac{(x-x^{2^k})(1+x^{2^k})}{(1-x)(1-x^{2^k})(1+x^{2^k})}$$