

1.108  
3

$$\textcircled{a} \sqrt{x - \frac{1}{x}} - \sqrt{1 - \frac{1}{x}} = \frac{x-1}{x} = 1 - \frac{1}{x}$$

$$\begin{array}{l} \begin{array}{c} + \quad + \\ 0 \quad 1 \end{array} \leftarrow 1 - \frac{1}{x} \geq 0 \\ \hline x < 0 \quad \vee \quad x \geq 1 \\ \begin{array}{c} + \quad + \\ -1 \quad 0 \quad 1 \end{array} \leftarrow x - \frac{1}{x} \geq 0 \\ \hline x \geq 1 \quad \vee \quad x < 0 \\ \hline (-1 \leq x < 0 \quad \vee \quad x \geq 1) \quad \text{or} \quad \emptyset \end{array}$$

מגבלות פתרון  
 $x \neq 0$

$$\sqrt{x - \frac{1}{x}} = \sqrt{1 - \frac{1}{x}} \left( 1 + \sqrt{1 - \frac{1}{x}} \right)$$

$$\sqrt{\frac{x^2-1}{x}} - \sqrt{\frac{x-1}{x}} \left( 1 + \sqrt{1 - \frac{1}{x}} \right) = 0$$

$$\sqrt{\frac{(x-1)(x+1)}{x}} - \sqrt{\frac{x-1}{x}} \left( 1 + \sqrt{1 - \frac{1}{x}} \right) = 0$$

$$\sqrt{\frac{x-1}{x}} \left( \sqrt{x+1} - 1 - \sqrt{1 - \frac{1}{x}} \right) = 0$$

$$\downarrow$$

$$x=1$$

$$\sqrt{x+1} = 1 + \sqrt{1 - \frac{1}{x}}$$

$$1 = \sqrt{x+1} - \sqrt{1 - \frac{1}{x}} \quad |(\ )^2$$

$$1 = x+1 - 2\sqrt{(x+1)\left(1 - \frac{1}{x}\right)} + 1 - \frac{1}{x} \rightarrow 0 = x+1 - \frac{1}{x} - 2\sqrt{x+1 - 1 - \frac{1}{x}}$$

$$0 = x+1 - \frac{1}{x} - 2\sqrt{x - \frac{1}{x}} \xrightarrow{\substack{A \\ \sqrt{x - \frac{1}{x}} = A}} 0 = A^2 + 1 - 2A = (A-1)^2 \rightarrow \boxed{A=1}$$

$$1 = \sqrt{x - \frac{1}{x}} \rightarrow 1 = x - \frac{1}{x} \rightarrow x^2 - x + 1 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

$\frac{1+\sqrt{5}}{2}$  : המספר הזה נקרא גולדן פובי

$x=1$      $\vee$      $x = \frac{1+\sqrt{5}}{2}$  : פתרון