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$$(3a-1)x^2 + 2ax + 3a-1 = 0 \rightarrow (3a-1)x^2 + 2ax + 3a-1 = 0$$

$$\textcircled{1} \rightarrow 0 \leq \Delta = 4a^2 - 4(3a-1)(3a-1) = 4a^2 - 4(9a^2 - 6a + 1)$$

$$0 \leq 4a^2 - 36a^2 + 24a - 4$$

$$32a^2 - 24a + 4 \leq 0 \quad /:4$$

$$8a^2 - 6a + 1 \leq 0$$

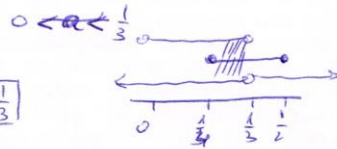


$$\boxed{\frac{1}{4} \leq a \leq \frac{1}{2}}$$

$$\textcircled{2} \quad \frac{c}{a} > 0 \quad \frac{c}{a} > 0 \text{ or } \Delta \geq 0 \text{ or } \text{for } a \neq 1$$

$$0 < \frac{c}{a} = \frac{3a-1}{3a-1} \rightarrow a \neq \frac{1}{3}$$

$$0 < \frac{b}{a} = \frac{-2a}{3a-1} \rightarrow \frac{1}{3} < a < \frac{1}{2}$$



$$\boxed{\frac{1}{4} \leq a < \frac{1}{3}}$$

$$\textcircled{3} \quad x_1 + x_2 = 3x_2 = \frac{-b}{a} = \frac{-2a}{3a-1} \rightarrow x_2 = \frac{-2a}{3(3a-1)}$$

$$x_1 \cdot x_2 = 2x_2^2 = \frac{3a-1}{3a-1} = 1$$

$$2 \left[\frac{-2a}{3(3a-1)} \right]^2 = 1$$

$$\left[\frac{-2a}{3(3a-1)} \right]^2 = \frac{1}{2} \quad / \sqrt{}$$

$$\frac{-2a}{3(3a-1)} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \quad \left| \quad \frac{-2a}{3(3a-1)} = -\sqrt{\frac{1}{2}} = -\frac{1}{\sqrt{2}} \right.$$

$$-2\sqrt{2}a = 9a - 3$$

$$a(9 + 2\sqrt{2}) = 3$$

$$a = \frac{3}{9 + 2\sqrt{2}}$$

$$a = \frac{3}{9 - 2\sqrt{2}}$$

in the first part of the problem, the condition $a \neq \frac{1}{3}$ is not needed

$$\textcircled{3} \quad x_1 + x_2 = \frac{-2a}{3a-1} \quad x_1 \cdot x_2 = 1$$

$$y_1 + y_2 = \frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{x_2^2 + x_1^2}{x_1^2 \cdot x_2^2} = \frac{(x_1 + x_2)^2 - 2x_1 x_2}{(x_1 x_2)^2} = \frac{4a^2}{(3a-1)^2 - 2}$$

$$= \frac{4a^2 - 2(9a^2 - 6a + 1)}{(3a-1)^2} = \frac{-14a^2 + 12a - 2}{(3a-1)^2}$$

$$y_1 y_2 = \frac{1}{x_1^2} \cdot \frac{1}{x_2^2} = \frac{1}{(x_1 x_2)^2} = 1$$

$$y^2 - \frac{-14a^2 + 12a - 2}{(3a-1)^2} y + 1 = 0$$

$$(3a-1)y^2 + (14a^2 - 12a + 2)y + (3a-1)^2 = 0$$

condition is not needed