

$$m(a_1 + a_n) = n(a_1 + a_m) \quad \leftarrow \quad \frac{S_m}{S_n} = \frac{m^2}{n^2} = \frac{\frac{m}{2}[a_1 + a_m]}{\frac{n}{2}[a_1 + a_n]} \quad \left(\frac{1}{2}\right)$$

$$ma_1 + ma_n = na_1 + na_m$$

$$a_1(m-n) + m(a_1 + d(n-1)) = n(a_1 + m(d-1))$$

$$a_1 m - a_1 n + ma_1 + mdn - md - na_1 - nm d - nd$$

$$2a_1 m - md = 2na_1 - nd$$

$$2a_1(2m - 2n) = d(m - n)$$

$$\boxed{2a_1 = d}$$

$$a_m = a_1 + d(m-1) = a_1 + 2a_1(m-1) = a_1(2m-1)$$

$$a_n = a_1 + d(n-1) = a_1 + 2a_1(n-1) = a_1(2n-1)$$

$$\frac{a_m}{a_n} = \frac{a_1(2m-1)}{a_1(2n-1)}$$