

2.76
22 (1) $\log_8 x_n - \log_8 x_{n+1} = \frac{1}{3} \rightarrow \log_8 \frac{x_n}{x_{n+1}} = \frac{1}{3} \rightarrow \frac{x_n}{x_{n+1}} = 8^{1/3} = 2$

103) $x_i = 2^{i-1} x_1$ $2x_n = x_{n+1}$

$$S = x_1 + x_1^2 x_2 + x_1^3 x_3 + x_1^4 x_4 + \dots$$

$$= x_1 + x_1^2 \cdot \frac{1}{2} x_1 + x_1^3 \cdot \frac{1}{4} x_1 + x_1^4 \cdot \frac{1}{8} x_1 + \dots$$

$$= x_1 + \frac{1}{2} x_1^3 + \frac{1}{4} x_1^4 + \frac{1}{8} x_1^5 + \dots$$

19) $x_1 = 1$ $x_2 = 2$ $x_3 = 4$ $x_4 = 8$ $x_5 = 16$ $x_6 = 32$ $x_7 = 64$ $x_8 = 128$ $x_9 = 256$ $x_{10} = 512$ $x_{11} = 1024$ $x_{12} = 2048$ $x_{13} = 4096$ $x_{14} = 8192$ $x_{15} = 16384$ $x_{16} = 32768$ $x_{17} = 65536$ $x_{18} = 131072$ $x_{19} = 262144$ $x_{20} = 524288$ $x_{21} = 1048576$ $x_{22} = 2097152$ $x_{23} = 4194304$ $x_{24} = 8388608$ $x_{25} = 16777216$ $x_{26} = 33554432$ $x_{27} = 67108864$ $x_{28} = 134217728$ $x_{29} = 268435456$ $x_{30} = 536870912$ $x_{31} = 1073741824$ $x_{32} = 2147483648$ $x_{33} = 4294967296$ $x_{34} = 8589934592$ $x_{35} = 17179869184$ $x_{36} = 34359738368$ $x_{37} = 68719476736$ $x_{38} = 137438953472$ $x_{39} = 274877906944$ $x_{40} = 549755813888$ $x_{41} = 1099511627776$ $x_{42} = 2199023255552$ $x_{43} = 4398046511104$ $x_{44} = 8796093022208$ $x_{45} = 17592186044416$ $x_{46} = 35184372088832$ $x_{47} = 70368744177664$ $x_{48} = 140737488355328$ $x_{49} = 281474976710656$ $x_{50} = 562949953421312$ $x_{51} = 1125899906842624$ $x_{52} = 2251799813685248$ $x_{53} = 4503599627370496$ $x_{54} = 9007199254740992$ $x_{55} = 18014398509481984$ $x_{56} = 36028797018963968$ $x_{57} = 72057594037927936$ $x_{58} = 144115188075855872$ $x_{59} = 288230376151711744$ $x_{60} = 576460752303423488$ $x_{61} = 1152921504606846976$ $x_{62} = 2305843009213693952$ $x_{63} = 4611686018427387904$ $x_{64} = 9223372036854775808$ $x_{65} = 18446744073709551616$ $x_{66} = 36893488147419103232$ $x_{67} = 73786976294838206464$ $x_{68} = 147573952589676412928$ $x_{69} = 295147905179352825856$ $x_{70} = 590295810358705651712$ $x_{71} = 1180591620717411303424$ $x_{72} = 2361183241434822606848$ $x_{73} = 4722366482869645213696$ $x_{74} = 9444732965739290427392$ $x_{75} = 18889465931478580854784$ $x_{76} = 37778931862957161709568$ $x_{77} = 75557863725914323419136$ $x_{78} = 151115727451828646838272$ $x_{79} = 302231454903657293676544$ $x_{80} = 604462909807314587353088$ $x_{81} = 1208925819614629174706176$ $x_{82} = 2417851639229258349412352$ $x_{83} = 4835703278458516698824704$ $x_{84} = 9671406556917033397649408$ $x_{85} = 19342813113834066795298816$ $x_{86} = 38685626227668133590597632$ $x_{87} = 77371252455336267181195264$ $x_{88} = 154742504910672534362390528$ $x_{89} = 309485009821345068724781056$ $x_{90} = 618970019642690137449562112$ $x_{91} = 1237940039285380274899124224$ $x_{92} = 2475880078570760549798248448$ $x_{93} = 4951760157141521099596496896$ $x_{94} = 9903520314283042199192993792$ $x_{95} = 19807040628566084398385987584$ $x_{96} = 39614081257132168796771975168$ $x_{97} = 79228162514264337593543950336$ $x_{98} = 158456325028528675187087900672$ $x_{99} = 316912650057057350374175801344$ $x_{100} = 633825300114114700748351602688$

$$-1 < \frac{1}{2} x_1 < 1$$

$$\boxed{-2 < x_1 < 2} \leftarrow$$

(2) $S = x_1 + \frac{x_1^2 \cdot x_2}{1 - \frac{1}{2} x_1} = \frac{1}{3} + \frac{\frac{1}{2} x_1^3}{1 - \frac{1}{2} x_1} = \frac{1}{3} + \frac{\frac{1}{2} \cdot (\frac{1}{3})^3}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{1}{3} + \frac{\frac{1}{54}}{\frac{5}{6}} = \frac{1}{3} + \frac{6}{5 \cdot 54} = \frac{1}{3} + \frac{1}{5 \cdot 9} = \frac{16}{45}$