

2.78

1

$$\textcircled{1} \begin{cases} a_1 + a_2 + \dots + a_n = S \\ a_1 q + a_2 q + \dots + a_n q = S q \end{cases} \rightarrow \begin{cases} S = a_1 + a_2 + \dots + a_n \\ S q = a_2 + a_3 + \dots + a_n q \end{cases}$$

$$S - S q = a_1 - a_n q = a_1 (1 - q^n)$$

$$S(1 - q) = a_1 (1 - q^n)$$

$$S = \frac{a_1 (1 - q^n)}{1 - q} = \frac{a_1 (q^n - 1)}{q - 1}$$

$$\textcircled{2} \begin{cases} S = 1 + 3x + 5x^2 + 7x^3 + \dots + (2n-1)x^{n-1} \\ Sx = x + 3x^2 + 5x^3 + 7x^4 + \dots + (2n-1)x^n \end{cases}$$

$$S - Sx = 1 + \underbrace{2x + 2x^2 + 2x^3 + \dots + 2x^{n-1}}_{+1023x \rightarrow 0} - (2n-1)x^n$$

$$S(1-x) = 1 + \frac{2x(x^{n-1}-1)}{x-1} - (2n-1)x^n$$

$$S = \frac{1}{1-x} - \frac{2x(x^{n-1}-1)}{(1-x)^2} - \frac{(2n-1)x^n}{1-x} = \frac{-2x(x^{n-1}-1) - [(2n-1)x^n-1](1-x)}{(1-x)^2}$$

$$= \frac{-2x^n + 2x + (2n-1)x^n(x-1) + 1 - x}{(1-x)^2} = \frac{-2x^n + x + 1 + (2n-1)x^n(x-1)}{(1-x)^2}$$