

2.19  
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$$\begin{aligned}x &= \cos a + \cos b \rightarrow x^2 = \cos^2 a + 2\cos a \cos b + \cos^2 b \quad + \\y &= \sin a + \sin b \rightarrow y^2 = \sin^2 a + 2\sin a \sin b + \sin^2 b\end{aligned}$$

אל תהיה  
מבולבל

$$\begin{aligned}x^2 + y^2 &= \cos^2 a + \sin^2 a + 2(\cos a \cos b + \sin a \sin b) + \cos^2 b + \sin^2 b \\x^2 + y^2 &= 1 + 2\cos(a-b) + 1 \\ \cos(a-b) &= \frac{x^2 + y^2}{2} - 1\end{aligned}$$

$$\begin{aligned}\cos(a-b) &= 2\cos^2\left(\frac{a-b}{2}\right) - 1 \\ \cos\left(\frac{a-b}{2}\right) &= \pm \sqrt{\frac{\cos(a-b) + 1}{2}} = \pm \sqrt{\frac{\frac{x^2 + y^2}{2} + 1}{2}} = \pm \frac{\sqrt{x^2 + y^2}}{2}\end{aligned}$$

המשפט הזה נקרא  
משפט וולקסטראד

$$\begin{aligned}x &= \cos a \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right) \\ x &= \pm 2 \frac{\sqrt{x^2 + y^2}}{2} \cdot \cos\left(\frac{a+b}{2}\right) \rightarrow \cos\left(\frac{a+b}{2}\right) = \pm \frac{x}{\sqrt{x^2 + y^2}}\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{a+b}{2}\right) &= \frac{-x}{\sqrt{x^2 + y^2}} & \cos\left(\frac{a+b}{2}\right) &= \frac{x}{\sqrt{x^2 + y^2}} \\ \cos\left(\frac{a-b}{2}\right) &= -\frac{\sqrt{x^2 + y^2}}{2} & \cos\left(\frac{a-b}{2}\right) &= \frac{\sqrt{x^2 + y^2}}{2}\end{aligned}$$