2.51

$n=1$

$$2\cos\alpha - 1 = \frac{2\cos(2\alpha)+1}{2\cos\alpha+1} = \frac{2(2\cos^2\alpha-1)+1}{2\cos\alpha+1} = \frac{4\cos^2\alpha-1}{2\cos\alpha+1} = \frac{(2\cos\alpha+1)(2\cos\alpha-1)}{2\cos\alpha+1} \checkmark$$

$n=k+1$

$$\underbrace{[2\cos\alpha-1]\cdots[2\cos(2^{n-1}\alpha)-1]}[2\cos(2^n\alpha)-1] \stackrel{?}{=} \frac{2\cos(2^{n+1}\alpha)+1}{2\cos\alpha+1}$$

$$\frac{2\cos(2^n\alpha)+1}{2\cos\alpha+1}\cdot 2\cos(2^n\alpha-1) \stackrel{?}{=}$$

$$\frac{4\cos^2(2^n\alpha)-1}{2\cos\alpha+1} = \frac{2\cdot(2\cos^2(2^n\alpha)-1)+1}{2\cos\alpha+1} = \frac{2\cos(2^{n+1}\alpha)+1}{2\cos\alpha+1}$$