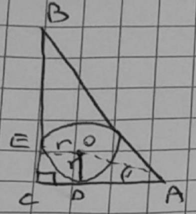
2.5 ג
דק



$\angle A_1 = \frac{1}{2}(90-\alpha) = 45 - \frac{\alpha}{2}$

EODC ריבוע (4 זוויות ישרות וזוג צלעות סמוכות שוות)

$CD = r$ $\qquad \frac{OD}{DA} = \tan(45 - \frac{\alpha}{2})$

$CD = r$ $\qquad DA = \frac{r}{\tan(45-\frac{\alpha}{2})}$ $\qquad :\triangle ODA$

במשולש $\triangle ABC$

$AC = 2R\sin\alpha \quad \Leftarrow \quad \frac{AC}{AB} = \sin\alpha$

$AC = CD + DA \Rightarrow \quad 2R\sin\alpha = r + \frac{r}{\tan(45-\frac{\alpha}{2})}$

$$2R\sin\alpha = r\left(1 + \frac{1}{\tan(45-\frac{\alpha}{2})}\right)$$

$$\frac{2\sin\alpha}{1+\frac{1}{\tan(45-\frac{\alpha}{2})}} = \frac{r}{R}$$

$$\frac{r}{R} = \frac{2\sin\alpha\tan(45-\frac{\alpha}{2})}{\tan(45-\frac{\alpha}{2})+1} = \frac{2\sin\alpha\left[\frac{1-\tan\frac{\alpha}{2}}{1+\tan\frac{\alpha}{2}}\right]}{\frac{1-\tan\frac{\alpha}{2}}{1+\tan\frac{\alpha}{2}}+1} =$$

$$= \frac{2\sin\alpha\,(1-\tan\frac{\alpha}{2})}{1-\tan\frac{\alpha}{2}+1+\tan\frac{\alpha}{2}} = \frac{2\sin\alpha\,(1-\tan\frac{\alpha}{2})}{2} = \sin\alpha\,(1-\tan\frac{\alpha}{2})$$

$$= 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\left(1-\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\right) = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right) = 2\sin\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)$$

$$= \sin\alpha - 2\sin^2\frac{\alpha}{2} = \sin\alpha + 1 - 2\sin^2\frac{\alpha}{2} - 1 = \sin\alpha + \cos\alpha - 1$$