

2.53
 $\frac{1}{2}M$

$n=1$ $\sin \frac{\pi}{3} = 2 \sin \frac{\pi}{6} \cos \frac{\pi}{3} = 2 \cdot \frac{1}{2} \cdot \cos \frac{\pi}{3} \checkmark$

$n=k+1$ $\sin \frac{\pi}{3} + \dots + \sin \frac{n\pi}{3} + \sin \frac{(n+1)\pi}{3} \stackrel{?}{=} 2 \sin \frac{(n+1)\pi}{6} \cos \left(\frac{n+2}{6} \pi \right)$

$2 \sin \frac{n\pi}{6} \cos \left(\frac{n+1}{6} \pi \right) + \sin \frac{(n+1)\pi}{3} \stackrel{?}{=}$

$2 \sin \frac{n\pi}{6} \cos \left(\frac{n+1}{6} \pi \right) + 2 \sin \frac{(n+1)\pi}{6} \cos \left(\frac{n+1}{6} \pi \right) \stackrel{?}{=}$

$2 \sin \frac{(n+1)\pi}{6} \left[\cos \frac{n\pi}{6} + \cos \frac{(n+1)\pi}{6} \right] \stackrel{?}{=}$

$2 \sin \frac{(n+1)\pi}{6} \left[\cos \left(\frac{\pi}{2} - \frac{n\pi}{6} \right) + \cos \frac{(n+1)\pi}{6} \right] \stackrel{?}{=}$

$2 \sin \frac{(n+1)\pi}{6} \cdot \underbrace{2 \cos \left(\frac{\pi}{4} + \frac{\pi}{12} \right)}_1 \cos \left(\frac{\pi}{4} - \frac{n\pi}{6} - \frac{\pi}{12} \right) = 2 \sin \frac{(n+1)\pi}{6} \cos \left(\frac{\pi}{6} - \frac{n\pi}{6} \right)$
 $2 \sin \frac{(n+1)\pi}{6} \cos \left(\frac{\pi}{6} - \frac{n\pi}{6} + \frac{n\pi}{6} \right)$