

2.54
p. 7

$$1 + \sin x + \cos 3x = \sin 2x + \cos x + \cos 2x$$

$$1 + \sin x - \sin 2x - \cos 2x + \cos 3x - \cos x = 0$$

$$1 + \sin x - 2\sin x \cos x - (1 - 2\sin^2 x) + 2\sin x \cos 2x = 0$$

$$\sin x - 2\sin x \cos x + 2\sin^2 x - 2\sin x \cos 2x = 0$$

$$\sin x - \sin 2x + 2\sin^2 x - 2\sin x \cos 2x = 0$$

$$\sin x (1 + 2\sin x) - \sin 2x (1 + 2\sin x) = 0$$

$$(\sin x - \sin 2x)(1 + 2\sin x) = 0$$

$$\downarrow$$
$$-2\sin \frac{x}{2} \cos 1.5x = 0$$

$$\frac{x}{2} = \pi k \rightarrow x = 2\pi k$$

$$1.5x = \frac{\pi}{2} + \pi k \rightarrow x = \frac{\pi}{3} + \frac{2\pi k}{3}$$

$$\rightarrow \sin x = -\frac{1}{2}$$

$$\boxed{x = -\frac{\pi}{6} + 2\pi k}$$
$$\boxed{x = \frac{7\pi}{6} + 2\pi k}$$