2.57

21) $n=1$ $\quad \frac{1}{\sin 2\alpha} = \operatorname{ctg}\alpha - \operatorname{ctg}2\alpha = \frac{\cos\alpha}{\sin\alpha} - \frac{\cos 2\alpha}{\sin 2\alpha} = \frac{\sin 2\alpha\cos\alpha - \sin\alpha\cos 2\alpha}{\sin\alpha\sin 2\alpha} =$

$$= \frac{\sin(2\alpha-\alpha)}{\sin\alpha\sin 2\alpha} = \frac{1}{\sin 2\alpha}$$

$n=k+1$

$$\underbrace{\frac{1}{\sin 2\alpha} + \dots + \frac{1}{\sin 2^n\alpha}} + \frac{1}{\sin 2^{n+1}\alpha} \overset{?}{=} \operatorname{ctg}\alpha - \operatorname{ctg}(2^{n+1}\alpha)$$

$$\cancel{\operatorname{ctg}\alpha} - \operatorname{ctg} 2^n\alpha + \frac{1}{\sin 2^{n+1}\alpha} \overset{?}{=} \cancel{\operatorname{ctg}\alpha} - \operatorname{ctg}(2^{n+1}\alpha)$$

$$\frac{-\cos 2^n\alpha}{\sin 2^n\alpha} + \frac{1}{\sin 2^{n+1}\alpha} \overset{?}{=} -\frac{\cos 2^{n+1}\alpha}{\sin 2^{n+1}\alpha}$$

$$-\frac{\cos 2^n\alpha}{\sin 2^n\alpha} \overset{?}{=} \frac{-1-\cos 2^{n+1}\alpha}{\sin 2^{n+1}\alpha} = \cancel{\frac{-1-(1-2\sin^2(2^n\alpha))}{\sin 2^{n+1}\alpha}}$$

$$\overset{?}{=} \frac{-1-(2\cos^2(2^n\alpha)-1)}{2\sin(2^n\alpha)\cos(2^n\alpha)}$$

$$\frac{-\cancel{2}\cos^{\cancel{2}}(2^n\alpha)}{\cancel{2}\sin(2^n\alpha)\cancel{\cos(2^n\alpha)}} = \frac{-\cos(2^n\alpha)}{\sin(2^n\alpha)} = -\operatorname{ctg}(2^n\alpha)$$