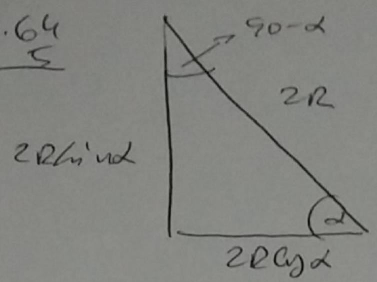
2.64
5



נחשב את שטח המשולש בשתי דרכים

ע"פ מכפלת הניצבים חלקי 2

$$2R^2\sin\alpha\cos\alpha$$

וע"פ הנוסחה rp

$$\frac{r(2R+2R\sin\alpha+2R\cos\alpha)}{2}$$

נשווה בין התוצאות

$$2R^2\sin\alpha\cos\alpha = Rr(1+\sin\alpha+\cos\alpha)$$

$$\frac{r}{R} = \frac{2\sin\alpha\cos\alpha}{1+\sin\alpha+\cos\alpha} = \frac{\sin 2\alpha}{1+\sqrt{2}\sin(\frac{\pi}{4}+\alpha)} \cdot \frac{1-\sqrt{2}\sin(\frac{\pi}{4}+\alpha)}{1-\sqrt{2}\sin(\frac{\pi}{4}+\alpha)} =$$

$$= \frac{\sin 2\alpha(1-\sqrt{2}\sin(\frac{\pi}{4}+\alpha))}{1-2\sin^2(\frac{\pi}{4}+\alpha)} = \frac{\sin 2\alpha(1-\sqrt{2}\sin(\frac{\pi}{4}+\alpha))}{\cos(2(\frac{\pi}{4}+\alpha))} =$$

$$= \frac{\sin 2\alpha(1-\sqrt{2}\sin(\frac{\pi}{4}+\alpha))}{\cos(\frac{\pi}{2}+2\alpha)} = \frac{\cancel{\sin 2\alpha}(1-\sqrt{2}\sin(\frac{\pi}{4}+\alpha))}{-\cancel{\sin 2\alpha}} =$$

$$= -1+\sin\alpha+\cos\alpha = \sin\alpha-(1-\cos\alpha) = \sin\alpha-2\sin^2\frac{\alpha}{2} =$$

$$= 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}-2\sin^2\frac{\alpha}{2} = 2\sin\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right) = 2\sin\frac{\alpha}{2}\left(\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right)$$

$$= 2\sqrt{2}\sin\frac{\alpha}{2}\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$