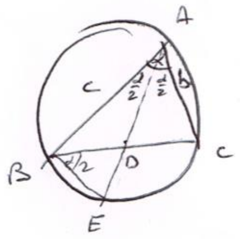
2.77
6

(א)



$\triangle ABC$:

$BC^2 = c^2+b^2-2bc\cdot\cos\alpha$

$\frac{BC}{\sin\alpha} = 2R$

$R = \frac{BC}{2\sin\alpha} = \frac{\sqrt{c^2+b^2-2bc\cdot\cos\alpha}}{2\sin\alpha}$

(ב)

$S = \frac{cb\cdot\sin\alpha}{2} = rp$

$r = \frac{cb\sin\alpha}{2p} = \frac{\cancel{2}cb\sin\alpha}{\cancel{2}(c+b+\sqrt{c^2+b^2-2bc\cdot\cos\alpha})} = \frac{cb\sin\alpha}{c+b+\sqrt{c^2+b^2-2bc\cdot\cos\alpha}}$

(ג)

נסמן $BD = x$ $\quad DC = \sqrt{c^2+b^2-2bc\cdot\cos\alpha} - x$

משפט חוצה זווית $\frac{BD}{CD} = \frac{AB}{AC} \rightarrow \frac{x}{\sqrt{c^2+b^2-2bc\cdot\cos\alpha} - x} = \frac{c}{b}$

$bx = c\sqrt{c^2+b^2-2bc\cdot\cos\alpha} - cx$

$x = \frac{c\sqrt{c^2+b^2-2bc\cdot\cos\alpha}}{b+c}$

$\triangle ABE$

$\frac{c}{\sin\angle E} = 2R = \frac{\sqrt{c^2+b^2-2bc\cdot\cos\alpha}}{\sin\alpha}$

$\sin\angle E = \frac{c\sin\alpha}{\sqrt{c^2+b^2-2bc\cdot\cos\alpha}}$

$\triangle BDE$:

$\frac{BD}{\sin\angle E} = \frac{DE}{\sin\frac{\alpha}{2}} \rightarrow DE = \frac{BD\sin\frac{\alpha}{2}}{\sin\angle E} =$

$DE = \frac{\frac{c\sqrt{c^2+b^2-2bc\cdot\cos\alpha}\,\sin\frac{\alpha}{2}}{b+c}}{\frac{c\sin\alpha}{\sqrt{c^2+b^2-2bc\cos\alpha}}} = \frac{\cancel{c}(c^2+b^2-2bc\cos\alpha)\sin\frac{\alpha}{2}}{(b+c)\cancel{c}\sin\alpha}$

$= \frac{(c^2+b^2-2bc\cdot\cos\alpha)\cancel{\sin\frac{\alpha}{2}}}{(b+c)2\cancel{\sin\frac{\alpha}{2}}\cos\frac{\alpha}{2}} = \frac{c^2+b^2-2bc\cdot\cos\alpha}{2(b+c)\cos\frac{\alpha}{2}}$