2.42
6

$rp = S$ (p = חצי היקף המשולש) נשתמש בנוסחת שטח משולש

$r(a+b+c) = 2S$

$r(a+b+c) = 4R^2 \sin\alpha \sin\beta \sin\gamma$

$2rR(\sin\alpha + \sin\beta + \sin\gamma) = 4R^2 \sin\alpha \sin\beta \sin\gamma$

$$\boxed{\frac{r}{R}} = \frac{2\sin\alpha\sin\beta\sin\gamma}{\sin\alpha+\sin\beta+\sin\gamma} = \frac{2\sin\alpha\sin\beta\sin(\alpha+\beta)}{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}} =$$

$$= \frac{2\cdot 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}\sin\alpha\sin\beta}{2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right)} = \frac{2\cos\frac{\alpha+\beta}{2}\,2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\,2\sin\frac{\beta}{2}\cos\frac{\beta}{2}}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}} = \boxed{4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}$$