

4.15
2

מציאת שורשי המשוואה $z^2 - 4z + 7 = 0$

$$(z - 2 - i\sqrt{3})(z - 2 + i\sqrt{3}) = z^2 - 4z + 7$$

$$\frac{z^2 - 6z^3 + 14z^2 - 10z - 7}{z^2 - 4z + 7}$$

$$z^4 - 4z^3 + 7z^2$$

$$-2z^3 + 7z^2 - 10z - 7$$

$$-2z^3 + 8z^2 - 14z$$

$$-z^2 + 4z - 7$$

$$-z^2 + 4z - 7$$

$$-$$

$$0 = z^2 - 4z + 7$$

$$z_{1,2} = \frac{4 \pm \sqrt{16 - 28}}{2} = 2 \pm i\sqrt{2}$$

4.15
5

$$\textcircled{a} z^3 = -\left(\frac{1+i\sqrt{3}}{-\sqrt{3}+i}\right)^3 = -\left(\frac{2\text{cis}60}{2\text{cis}150}\right)^3 = -(\text{cis}(-90))^3 = \text{cis}180 \cdot \text{cis}(-270) = 1$$

$$z^3 = -1 = \text{cis}270 \rightarrow z = \text{cis}\left(\frac{2\pi k}{3}\right) \quad k=0,1,2$$

$$\textcircled{b} z = \frac{i^{1434}(1+i)^8}{2(\sin\frac{5\pi}{12} - i\cos\frac{5\pi}{12})} = \frac{i^3(\sqrt{2}\text{cis}45)^8}{2(\cos(\frac{\pi}{2} - \frac{5\pi}{12}) + i\sin(\frac{\pi}{2} - \frac{5\pi}{12}))} = \frac{\text{cis}270 \cdot 2^4 \cdot \text{cis}0}{2(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12})} =$$

$$= \frac{2^4 \text{cis}270}{2(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12}))} = \frac{2^3 \text{cis}270}{\text{cis}(-\frac{\pi}{12})} = \frac{2^3 \text{cis}(\frac{3\pi}{2})}{\text{cis}(-\frac{\pi}{12})} = 2^3 \text{cis}\left(\frac{19\pi}{12}\right)$$

$$|z| = 2^3 = 8, \quad \arg z = \frac{19\pi}{12}$$