4.25 סעיף 9

(1) $z^5 = -1 \quad (\cdot)^{20}$

$z^{100} = 1$

$(z_1)^{101} + (z_2)^{101} + (z_3)^{101} + (z_4)^{101} + (z_5)^{101} =$

$z_1 \cdot z_1^{100} + z_2 \cdot z_2^{100} + z_3 \cdot z_3^{100} + z_4 \cdot z_4^{100} + z_5 \cdot z_5^{100} =$

$z_1 + z_2 + z_3 + z_4 + z_5 = 0$

(2) $z^5 = -1 = \text{cis}\,180 \rightarrow z_k = \text{cis}\left(\frac{\pi}{5} + \frac{2\pi k}{5}\right), \quad k = 0,1,2,3,4$

$w = \text{cis}\left(\frac{3\pi}{5}\right)$ נציב $k=1$ ונקבל

$$\left|w^5 + w^2\right|^2 = \left|\left(\text{cis}\left(\tfrac{3\pi}{5}\right)\right)^5 + \left(\text{cis}\left(\tfrac{3\pi}{5}\right)\right)^2\right|^2 = \left|\text{cis}(3\pi) + \text{cis}\left(\tfrac{6\pi}{5}\right)\right|^2$$

$$= \left|-1 + \text{cis}\left(\tfrac{6\pi}{5}\right)\right|^2 = \left(\cos\left(\tfrac{6\pi}{5}\right) - 1\right)^2 + \sin^2\left(\tfrac{6\pi}{5}\right) = \cos^2\left(\tfrac{6\pi}{5}\right) - 2\cos\left(\tfrac{6\pi}{5}\right) +$$

$$1 + \sin^2\left(\tfrac{6\pi}{5}\right)) = 2 - 2\cos\left(\tfrac{6\pi}{5}\right) = 2\left(1 - \cos\left(\tfrac{6\pi}{5}\right)\right) = 2 \cdot 2\sin^2\left(\tfrac{3\pi}{5}\right) =$$

$$= 4\sin^2\left(\tfrac{3\pi}{5}\right)$$