

3.52
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$$\textcircled{1} z_1 = r_1 \text{cis } \alpha \quad z_2 = r_2 \text{cis } \beta$$

$$\arg z_1 - \arg z_2 = \alpha - \beta$$

$$\arg \left(\frac{z_1}{z_2} \right) = \arg \left(\frac{r_1 \text{cis } \alpha}{r_2 \text{cis } \beta} \right) = \alpha - \beta$$

$$\textcircled{2} \left| \frac{x^2 - y^2 + 2xyi}{\sqrt{2}xy + i\sqrt{x^4 + y^4}} \right| = \frac{|x^2 - y^2 + 2xyi|}{|\sqrt{2}xy + i\sqrt{x^4 + y^4}|} = \frac{\sqrt{(x^2 - y^2)^2 + (2xy)^2}}{((\sqrt{2}xy)^2 + (\sqrt{x^4 + y^4})^2)^{1/2}}$$
$$= \sqrt{\frac{x^4 - 2x^2y^2 + y^4 + 4x^2y^2}{2x^2y^2 + x^4 + y^4}} = \sqrt{\frac{x^4 + 2x^2y^2 + y^4}{x^4 + 2x^2y^2 + y^4}} = 1$$

$$\textcircled{3} 4 \geq |z - i| = |x + iy - i| = \sqrt{x^2 + (y-1)^2} \rightarrow 16 \geq x^2 + (y-1)^2$$

$$1 \geq \operatorname{Re} z^2 = \operatorname{Re}(x^2 + 2xyi - y^2) \rightarrow 1 \geq x^2 - y^2 \quad (\text{standard})$$

