

3.54
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א) $z_1 = 2 - i\sqrt{3}$ $z_2 = 2 + i\sqrt{3}$: z_1, z_2 הם שורשי המשוואה $x^2 - 4x + 7 = 0$

$$(x - z_1)(x - z_2) = (x - 2 + i\sqrt{3})(x - 2 - i\sqrt{3}) = x^2 - 4x + 7$$

$$\begin{array}{r} x^2 - 3 \\ x^4 - 4x^3 + 4x^2 + 12x - 21 \quad | \quad x^2 - 4x + 7 \\ \hline x^4 - 4x^3 + 7x^2 \\ \hline 3x^2 - 12x - 21 \\ -3x^2 + 12x - 21 \\ \hline 0 \end{array}$$

פירוש: $x = \pm\sqrt{3}$; אכן

ב) $|z - \bar{z}| = |z| - 4 = 0$

$$i(x + iy) - (x - iy) + \sqrt{x^2 + y^2} - 4 = 0$$

$$x + y - x + iy + \sqrt{x^2 + y^2} - 4 = 0$$

$$i(x + y) + (-y - x + \sqrt{x^2 + y^2} - 4) = 0$$

i) $x + y = 0 \rightarrow x = -y$

אבל: $|z| = 4$; אכן

ii) $-y + x + \sqrt{x^2 + y^2} - 4 = 0 \rightarrow \sqrt{2x^2} = 4 \rightarrow x^2 = 8 \rightarrow x = \pm\sqrt{8}$
 $z_{1,2} = \pm\sqrt{8}(1 - i) = \pm 2\sqrt{2}(1 - i)$