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$$I \quad 1 + i \tan \alpha = \frac{\cos \alpha + i \sin \alpha}{\cos \alpha} = \frac{cis \alpha}{\cos \alpha}$$

$$1 - i \tan \alpha = \frac{\cos \alpha - i \sin \alpha}{\cos \alpha} = \frac{\cos(\alpha) + i \sin(\alpha)}{\cos \alpha} = \frac{cis(-\alpha)}{\cos \alpha}$$

$$z = \left(\frac{1 + i \tan \alpha}{1 - i \tan \alpha} \right)^3 = \left(\frac{\frac{cis \alpha}{\cos \alpha}}{\frac{cis(-\alpha)}{\cos \alpha}} \right)^3 = (cis 2\alpha)^3 = cis 6\alpha$$

$$\theta = 6\alpha, \quad r = 1$$

$$II \quad \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)^3 = \left(\frac{1 + i \tan 60}{1 - i \tan 60} \right)^3 = cis 360 \quad \text{znk } \frac{\rho}{\rho} \text{ o'k}$$

$$z^4 = -cis 360 = -1 = cis 180 = cis \pi$$

$$z_k = cis \left(\frac{\pi}{4} + \frac{\pi k}{2} \right) \quad k=0,1,2,3$$

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$$I \quad z = 1 + \cos \varphi + i \sin \varphi$$

$$|z| = \sqrt{(1 + \cos \varphi)^2 + \sin^2 \varphi} =$$

$$= \sqrt{1 + 2\cos \varphi + \cos^2 \varphi + \sin^2 \varphi} = \sqrt{2 + 2\cos \varphi} =$$

$$= \sqrt{2 \cdot (1 + \cos \varphi)} = \sqrt{2 \cdot 2 \cos^2 \frac{\varphi}{2}} = 2 \cos \frac{\varphi}{2}$$

$$\arg z: \quad \tan \alpha = \frac{\sin \varphi}{1 + \cos \varphi} = \frac{\sin \varphi}{2 \cos^2 \frac{\varphi}{2}} = \frac{2 \sin \frac{\varphi}{2} \cos \frac{\varphi}{2}}{2 \cos^2 \frac{\varphi}{2}} = \tan \frac{\varphi}{2}$$

$$\alpha = \frac{\varphi}{2}$$

$$II \quad z^6 = 2 \cos \frac{60}{2} \cdot cis \frac{60}{2} = 2 \cos 30 \cdot cis 30 = \sqrt{3} cis 30$$

$$z^8 = 3^4 cis 240 = 81 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i \right) = -\frac{81}{2} (1 + \sqrt{3} i)$$

$$\sqrt[3]{z^8} = \sqrt[3]{-\frac{81}{2} (1 + \sqrt{3} i)} = \sqrt[3]{\sqrt{3} cis 30} = \sqrt[6]{3} cis (10 + 120k) \quad k=0,1,2$$