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$$1 + i \tan \alpha = \frac{\cos \alpha + i \sin \alpha}{\cos \alpha} = \frac{\operatorname{cis} \alpha}{\cos \alpha}$$

$$1 - i \tan \alpha = \frac{\cos \alpha - i \sin \alpha}{\cos \alpha} = \frac{\operatorname{cis}(-\alpha)}{\cos \alpha}$$

$$\frac{1 + i \tan \alpha}{1 - i \tan \alpha} = \frac{\frac{\operatorname{cis} \alpha}{\cos \alpha}}{\frac{\operatorname{cis}(-\alpha)}{\cos \alpha}} = \operatorname{cis} 2\alpha$$

$$\frac{1 + xi}{1 - xi} = \operatorname{cis} 2\beta \quad | \text{ mit } x = \tan \beta \quad \text{mit } \beta \text{ mit } 2\beta$$

$$(\operatorname{cis} 2\beta)^{10} = \operatorname{cis} 2\alpha$$

$$\operatorname{cis} 20\beta = \operatorname{cis} 2\alpha$$

$$\beta = \frac{2\alpha}{20} + \frac{2\pi k}{20} = \frac{\alpha}{10} + \frac{\pi k}{10} \quad k=0,1,\dots,9$$

$$x = \tan \beta = \tan\left(\frac{\alpha}{10} + \frac{\pi k}{10}\right) \quad | \text{ mit}$$

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$$3 \geq |z-1| \rightarrow 9 \geq x^2 + (y-1)^2$$

$$-1 < \operatorname{Im} z \leq 2 \rightarrow -1 < y \leq 2$$

$$\frac{1}{z} \geq \operatorname{Im} \frac{1}{z} = \operatorname{Im}\left(\frac{1}{x+iy} \cdot \frac{x-iy}{x-iy}\right) = \operatorname{Im}\left(\frac{x-iy}{x^2+y^2}\right) = \frac{-y}{x^2+y^2}$$

$$x^2+y^2 \geq -2y \rightarrow x^2+(y+1)^2 \geq 1$$

