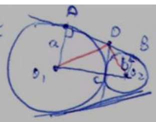
3.28
5

א.



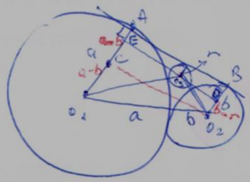
DO_1 חוצה זווית $\measuredangle ADC$ ⇐ ΔO_1DO_2 משלים עד זווית

DO_2 " " $\measuredangle CDB$

$DC \perp O_1O_2$ ⇐ $\Delta DO_1C \sim \Delta O_2DC$ (s.s)

$\frac{DC}{O_1C} = \frac{O_2C}{DC} \rightarrow DC^2 = O_1C \cdot O_2C \rightarrow DC = \sqrt{ab}$

(2 משיקים היוצאים מנק' אחת...) $AD = DC = DB \rightarrow AB = 2DC = 2\sqrt{ab}$



דרך נוספת: המרובע ABO_2C מלבן

ΔO_1CO_2: $O_1O_2^2 = O_1C^2 + CO_2^2$

$(a+b)^2 = (a-b)^2 + CO_2^2$

$CO_2 = AB = \sqrt{4ab} = 2\sqrt{ab}$

ב.

ΔO_3O_2D: $O_3O_2^2 = O_3D^2 + DO_2^2$

$(r+b)^2 = O_3D^2 + (b-r)^2$

$O_3D = 2\sqrt{rb}$

ΔO_3EO_1: $O_1O_3^2 = O_3E^2 + O_1E^2$

$(r+a)^2 = O_3E^2 + (a-r)^2 \rightarrow O_3E = 2\sqrt{ra}$

$O_3D + O_3E = ED = AB = 2\sqrt{ab}$

$2\sqrt{ra} + 2\sqrt{rb} = 2\sqrt{ab}$

$\sqrt{r}(\sqrt{a} + \sqrt{b}) = \sqrt{ab} \rightarrow r = \frac{ab}{(\sqrt{a}+\sqrt{b})^2}$