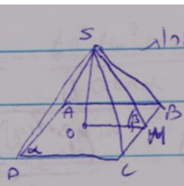
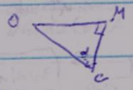
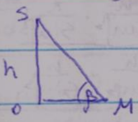
3.98
8

א. סכום 2 צלעות נגדיות שווה לסכום 2 הצלעות האחרות



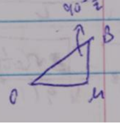




ב. נסמן $SO=h$

$SM=\frac{h}{\sin\beta}$

$OM=\frac{h}{\tan\beta}$

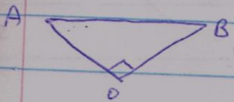


$OC=\frac{OM}{\sin\frac{\alpha}{2}}=\frac{h}{\sin\frac{\alpha}{2}\tan\beta}\to AC=2OC=\frac{2h}{\sin\frac{\alpha}{2}\tan\beta}$

$OB=\frac{OM}{\sin(90-\frac{\alpha}{2})}=\frac{h}{\cos\frac{\alpha}{2}\tan\beta}\to BD=2OB=\frac{2h}{\cos\frac{\alpha}{2}\tan\beta}$

$$V=\frac{1}{3}\cdot \underset{\text{בסיס הפירמידה}}{S}\cdot \overset{\text{קודקוד הפירמידה}}{SO}=\frac{1}{3}\cdot\frac{AC\cdot BD\cdot SO}{2}=\frac{1}{3}\cdot\frac{4h^2}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan^2\beta}\cdot h$$

$$V=\frac{4h^3}{3\sin\alpha\tan^2\beta}\longrightarrow h=\sqrt[3]{\frac{3V\sin\alpha\tan^2\beta}{4}}$$

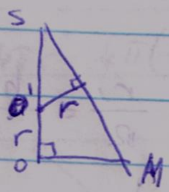


$$AB=\sqrt{AO^2+BO^2}=\frac{h}{\tan\beta}\sqrt{\frac{1}{\sin^2\frac{\alpha}{2}}+\frac{1}{\cos^2\frac{\alpha}{2}}}=\frac{h}{\tan\beta}\sqrt{\frac{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}}}$$

$$=\frac{h}{\tan\beta}\sqrt{\frac{4}{\sin^2\alpha}}=\frac{2h}{\tan\beta\sin\alpha}$$

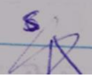
$$AB=\frac{2}{\tan\beta\sin\alpha}\sqrt[3]{\frac{3V\sin\alpha\tan^2\beta}{4}}=\sqrt[3]{\frac{6V}{\sin^2\alpha\tan\beta}}$$

ג



נסתכל על החתך SOM

$$\frac{r}{OM}=\frac{SO'}{SM}\Rightarrow\frac{SO-r}{SM}$$



$$rSA=OA\cdot SO-r\cdot OA$$

$$r=\frac{MO\cdot SM}{SM+OM}=\frac{\frac{h}{\tan\beta}\cdot\frac{h}{\sin\beta}}{\frac{h}{\tan\beta}+\frac{h}{\sin\beta}}=\frac{\frac{h^2}{\tan\beta\sin\beta}}{h\left(\frac{\sin\beta+\tan\beta}{\sin\beta\tan\beta}\right)}=\frac{h}{\sin\beta+\tan\beta}$$

$$=\frac{1}{\sin\beta+\tan\beta}\sqrt[3]{\frac{3V\sin\alpha\tan^2\beta}{4}}=\frac{1}{\sin\beta+\frac{\sin\beta}{\cos\beta}}\sqrt[3]{\frac{3V\sin\alpha\tan^2\beta}{4}}=\frac{\cos\beta}{\sin\beta\cos\beta+\sin\beta}\sqrt[3]{\frac{3V\sin\alpha\tan^2\beta}{4}}=$$

$$=\frac{\cos\beta}{\sin\beta(1+\cos\beta)}\sqrt[3]{\frac{3V\sin\alpha\tan^2\beta}{4}}=\sqrt[3]{\frac{\cos^3\beta\cdot 3V\sin\alpha\tan^2\beta}{4\sin^3\beta(1+\cos\beta)^3}}=\sqrt[3]{\frac{\cos\beta}{\sin\beta}\cdot\frac{3V\sin\alpha}{4(1+\cos\beta)^3}}=\sqrt[3]{\frac{3V\cot\beta\sin\alpha}{4(1+\cos\beta)^3}}$$

נשתמש בזהות $\tan\frac{1}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

$$\sqrt[3]{\frac{3V\cot\beta\sin\alpha}{4(1+\cos\beta)^3}\cdot\frac{\sin^3\beta}{\sin^3\beta}} = \sqrt[3]{\frac{3V\cot\beta\sin\alpha}{4}}\cdot\frac{\tan\frac{\beta}{2}}{\sin\beta} = \frac{1}{2}\frac{\tan\frac{\beta}{2}}{\sin\beta}\sqrt[3]{6V\cot\beta\sin\alpha}$$