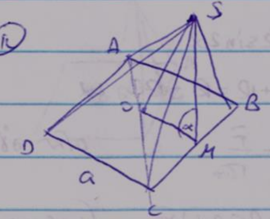
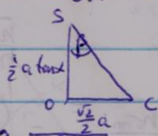
3.99
8

(1)



$AC = \sqrt{2}a \qquad OM = \frac{1}{2}a$

$\frac{SO}{OM} = \tan\alpha \rightarrow SO = \frac{1}{2}a\tan\alpha$



$SC = \sqrt{\frac{1}{4}a^2\tan^2\alpha + \frac{2}{4}a^2} = \frac{a}{2}\sqrt{\tan^2\alpha+2}$

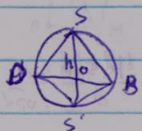
$\cos\beta = \dfrac{\frac{1}{2}a\tan\alpha}{\frac{a}{2}\sqrt{\tan^2\alpha+2}} = \dfrac{\tan\alpha}{\sqrt{\tan^2\alpha+2}}$

$\sin\beta = \dfrac{\frac{\sqrt{2}}{2}a}{\frac{a}{2}\sqrt{\tan^2\alpha+2}} = \sqrt{\dfrac{2}{\tan^2\alpha+2}}$

$\sin\angle ASC = \sin 2\beta = 2\sin\beta\cos\beta = 2\dfrac{\tan\alpha}{\sqrt{\tan^2\alpha+2}}\sqrt{\dfrac{2}{\tan^2\alpha+2}} = \dfrac{2\sqrt{2}\tan\alpha}{2+\tan^2\alpha} =$

$= \dfrac{\frac{2\sqrt{2}}{\cot\alpha}}{2+\frac{1}{\cot^2\alpha}} = \dfrac{2\sqrt{2}\cot\alpha}{1+2\cot^2\alpha}$

(2)



נסתכל על החתך המעגלי של הכדור המכיל את SB

$SO \cdot OS' = B'O \cdot OD \rightarrow h(2R-h) = OB^2$

$2Rh = h^2 + OB^2 \rightarrow R = \dfrac{h^2+OB^2}{2h}$

$R = \dfrac{\frac{1}{4}a^2\tan^2\alpha + \frac{a^2}{2}}{a\tan\alpha} = \dfrac{a^2\tan^2\alpha + 2a^2}{4a\tan\alpha} = \dfrac{a(\tan^2\alpha+2)}{4\tan\alpha} =$

$\dfrac{a\left(\frac{1}{\cos^2\alpha}+1\right)}{\frac{4\sin\alpha}{\cos\alpha}} = \dfrac{a(1+\cos^2\alpha)}{4\sin\alpha\cos\alpha} = \dfrac{a(1+\cos^2\alpha)}{2\sin 2\alpha} = \dfrac{a(2\cos^2\alpha+\sin^2\alpha)}{2\sin 2\alpha} =$

~~$= \dfrac{a(\cos^2\alpha+1+\sin^2\alpha)}{2\sin 2\alpha}$~~

$= \dfrac{2a(1+\cos^2\alpha)}{4\sin 2\alpha} = \dfrac{a(2+2\cos^2\alpha)}{4\sin 2\alpha} =$

$= \dfrac{a(3+\cos 2\alpha)}{4\sin 2\alpha}$