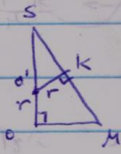
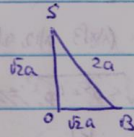
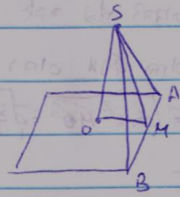


4.19
6



$$SM = \sqrt{SO'^2 + O'M^2} = \sqrt{2r^2 + r^2} = \sqrt{3}r$$

$$\frac{SO'}{SM} = \frac{O'M}{OM}$$

$$\frac{SO - r}{\sqrt{3}r} = \frac{r}{a} \rightarrow SO - r = \sqrt{3}r \Rightarrow SO = r(1 + \sqrt{3})$$

$$r = \frac{\sqrt{2}a}{1 + \sqrt{3}}$$

$$S_{\text{pyr}} = 4\sqrt{3}r^2 = 4\sqrt{3} \left(\frac{\sqrt{2}a}{1 + \sqrt{3}} \right)^2 = 4\sqrt{3} \cdot \frac{2a^2}{4 + 2\sqrt{3}} = \frac{4\sqrt{3}a^2}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = 4\sqrt{3}a^2(2 - \sqrt{3})$$