3.7.4

1.א $\sin^2\alpha + \sin\alpha \cdot \sin 3\alpha + \sin\alpha \cdot \sin 5\alpha + \dots + \sin\alpha \cdot \sin(2n-1)\alpha$

$= \sin^2 n\alpha$

אין צורך אפיונים

נוכיח ל-n+1:

$\underbrace{\sin^2\alpha + \dots + \dots +}_{\text{הנחה}} \sin\alpha \cdot \sin(2n+1)\alpha = \sin^2(n\alpha+\alpha)$

$\sin^2 n\alpha + \sin\alpha \sin(2n+1)\alpha = \sin^2(n\alpha+\alpha)$

$\sin\alpha \sin(2n+1)\alpha = \sin^2(n\alpha+\alpha) - \sin^2(n\alpha)$

$— \, // \, — = [\sin(n\alpha+\alpha) - \sin(n\alpha)] \cdot [\sin(n\alpha+\alpha) + \sin(n\alpha)]$

$— // — = \left[2\sin\frac{\alpha}{2}\cos\frac{2n\alpha+\alpha}{2}\right]\left[2\sin\frac{2n\alpha+\alpha}{2}\cos\frac{\alpha}{2}\right]$

$\cancel{\sin\alpha} \sin(2n+1)\alpha = \cancel{\sin\alpha} \cdot \left(\cos\frac{2n\alpha+\alpha}{2} \cdot 2\sin\frac{2n\alpha+\alpha}{2}\right)$

$\sin(2n+1)\alpha = 2\cos\frac{\alpha(2n+1)}{2} \cdot \sin\frac{\alpha(2n+1)}{2}$

$\sin(2n+1)\alpha = \sin\alpha(2n+1)$ ✓