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$n=k+1$

$$\frac{1}{k+3} + \frac{1}{k+4} + \dots + \frac{1}{3k+1} + \frac{1}{3k+2} + \frac{1}{3k+3} + \frac{1}{3k+4} \stackrel{?}{>} \frac{k+1}{k+2}$$

$$\frac{k}{k+1} - \frac{1}{k+2} + \frac{1}{3k+2} + \frac{1}{3(k+1)} + \frac{1}{3k+4} \stackrel{?}{>} \frac{k+1}{k+2}$$

$$\frac{3k+1}{3(k+1)} + \frac{3k+4+3k+2}{(3k+2)(3k+4)} \stackrel{?}{>} \frac{k+1+1}{k+2}$$

$$\frac{6k+6}{(3k+2)(3k+4)} \stackrel{?}{>} 1 - \frac{3k+1}{3k+3}$$

$$\frac{6(k+1)}{9k^2+18k+8} > \frac{3k+3-3k-1}{3(k+1)}$$

$$18(k+1)^2 > 2(9k^2+18k+8)$$

$$9k^2+18k+9 > 9k^2+18k+8 \quad \checkmark$$