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$$a^{\log x} = x^{\log a} \quad \text{למה? (אם $x > 0$)}$$

$$- \log x \log a = \log a \log x \quad \text{למה? (אם $x > 0$)}$$

$$\begin{cases} (2x)^{\log 2} = (3y)^{\log 3} \\ 3^{\log x} = 2^{\log y} \end{cases}$$

המשוואה השנייה

$$x^{\log 3} = y^{\log 2}$$

$$x = \sqrt[\log 3]{y^{\log 2}} = y^{\frac{\log 2}{\log 3}}$$

המשוואה הראשונה

$$\left(2y^{\frac{\log 2}{\log 3}}\right)^{\log 2} = (3y)^{\log 3}$$

$$2^{\log 2} \cdot y^{\frac{\log 2^2}{\log 3}} = 3^{\log 3} \cdot y^{\log 3}$$

$$y^{\frac{\log 2^2}{\log 3} - \log 3} = \frac{3^{\log 3}}{2^{\log 2}}$$

למה? (אם $x > 0$)

$$\left(\frac{\log 2^2}{\log 3} - \log 3\right) \log y = \log \left(\frac{3^{\log 3}}{2^{\log 2}}\right)$$

$$\frac{\log 2^2 - \log 3^2}{\log 3} \cdot \log y = \log(3^{\log 3}) - \log(2^{\log 2})$$

$$\frac{\log 2^2 - \log 3^2}{\log 3} \cdot \log y = \log 3^2 - \log 2^2$$

$$\log y = -\log 3 = \log \frac{1}{3} \rightarrow \boxed{y = \frac{1}{3}}$$

$$x^{\log 3} = y^{\log 2}$$

$$\log 3 \log x = \log y \cdot \log 2 = \log \frac{1}{3} \cdot \log 2 = -\log 3 \log 2 \rightarrow \boxed{x = \frac{1}{2}}$$

