3.76
2

$$2\log_x 3 \cdot \log_{3x} 3 \geq \log_{9\sqrt{x}} 3$$

תחום הגדרה

$0 < x \neq 1$

$0 < 3x \neq 1$

$0 < 9\sqrt{x} \neq 1$

$\rightarrow \quad 0 < x \neq 1, \frac{1}{3}, \frac{1}{81}$

נעבור לבסיס משותף 3:

$$\frac{2\log_3 3}{\log_3 x} \cdot \frac{\log_3 3}{\log_3 3x} \geq \frac{\log_3 3}{\log_3 9\sqrt{x}}$$

$$\frac{2}{\log_3 x} \cdot \frac{1}{\log_3 3 + \log_3 x} \geq \frac{1}{\log_3 9 + \log_3 \sqrt{x}}$$

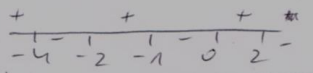
נסמן $\log_3 x = t$

$$\frac{2}{t} \cdot \frac{1}{1+t} \geq \frac{1}{2+\frac{1}{2}t}$$

$$\frac{2}{t(1+t)} \geq \frac{2}{4+t}$$

$$0 \leq 2\left(\frac{1}{t(1+t)} - \frac{1}{4+t}\right) = 2\left(\frac{4+t-t-t^2}{t(1+t)(4+t)}\right) = -2\left(\frac{t^2-4}{t(1+t)(4+t)}\right)$$

$$0 \leq -2\,\frac{(t+2)(t-2)}{t(1+t)(4+t)}$$



$0 < t \leq 2 \quad \rightarrow \quad 0 < \log_3 x < 2 \quad \rightarrow \quad 1 < x \leq 9$

$-2 \geq t < -1 \quad \rightarrow \quad -2 < \log_3 x < -1 \quad \rightarrow \quad \frac{1}{9} \leq x < \frac{1}{3}$

$t < -4 \quad \rightarrow \quad \log_3 x < -4 \quad \rightarrow \quad 0 < x < \frac{1}{81}$

תחום הגדרה