

4.16  
3

$$\log_{3x} x + \log_3^2 x < \frac{1}{\log_x 3}$$
$$\frac{\log_3 x}{\log_3 3x} + \log_3^2 x < \log_3 x$$

0 < x < 1  
1 ≠ x > 0

$$\frac{\log_3 x}{\log_3 3 + \log_3 x} + \log_3^2 x < \log_3 x$$

$$\frac{\log_3 x}{1 + \log_3 x} + \log_3^2 x < \log_3 x$$

$$\frac{t}{1+t} + t^2 < t$$

$$\log_3 x = t$$

$$\frac{t + t^2 + t^3 - t - t^2}{1+t} < 0$$

$$\frac{t^3}{1+t} < 0$$

$$\begin{array}{c} + \quad + \\ \hline -1 \quad 0 \end{array}$$

$$-1 < t < 0$$

$$-1 < \log_3 x < 0$$

$$\frac{1}{3} < x < 1$$