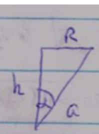
4.21
6

(א)



$h = a\cos\alpha \qquad , R = a\sin\alpha$

(ב) נסמן את רדיוס החתך העליון ב- R, רדיוס החתך השני ב- r ורדיוס החתך האמצעי ב- $\bar{r}$.

$$\begin{cases} \dfrac{x}{h} = \dfrac{\bar{r}}{R} \\ \dfrac{h-x}{h} = \dfrac{\bar{r}}{r} \end{cases} \qquad R = a\sin\alpha$$

$r = h\tan\beta = a\cos\alpha\tan\beta$

$$\begin{cases} Rx = h\bar{r} \\ hr - xr = h\bar{r} \end{cases} \rightarrow Rx = hr - xr$$

$$x = \frac{hr}{R+r} = \frac{r\,a\cos\alpha}{a\sin\alpha + a\cos\alpha\tan\beta} = \frac{r\cos\alpha}{\sin\alpha + \cos\alpha\tan\beta} =$$

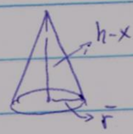
$$= \frac{a\cos^2\alpha\tan\beta}{\sin\alpha + \cos\alpha\tan\beta}$$

$$\bar{r} = \frac{Rx}{h} = \frac{a\sin\alpha \cdot a\cos^2\alpha\tan\beta}{a\cos\alpha(\sin\alpha + \cos\alpha\tan\beta)} = \frac{a\sin\alpha\tan\beta\cos\alpha}{\sin\alpha + \cos\alpha\tan\beta}$$

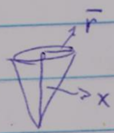
$$S = \pi\bar{r}^2 = \frac{\pi a^2\sin^2\alpha\tan^2\beta\cos^2\alpha}{(\sin\alpha + \cos\alpha\tan\beta)^2} = \frac{\pi a^2\sin^2\alpha\cos^2\alpha\sin^2\beta}{\cos^2\beta\dfrac{(\sin\alpha\cos\beta + \cos\alpha\sin\beta)^2}{\cos^2\beta}} =$$

$$= \frac{\pi a^2\sin^2\alpha\cos^2\alpha\sin^2\beta}{\sin^2(\alpha+\beta)}$$

(ג)



החלק המשולש העליון



החלק המשולש התחתון

$$V_1 + V_2 = \frac{1}{3}\pi\bar{r}^2(h-x) + \frac{1}{3}\pi\bar{r}^2 x = \frac{1}{3}\pi\bar{r}^2 h =$$

$$= \frac{1}{3}\pi\,\frac{a^2\sin^2\alpha\cos^2\alpha\sin^2\beta}{\sin^2(\alpha+\beta)} \cdot a\cos\alpha = \frac{\pi a^3\sin^2\alpha\cos^3\alpha\sin^2\beta}{3\sin^2(\alpha+\beta)}$$