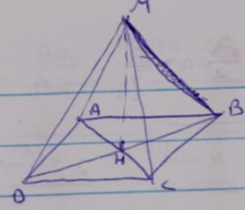
3,15
8

(א)



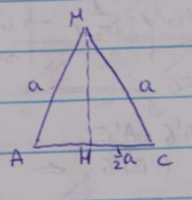
$\triangle MAC$ משולש שווה צלעות

$\Leftarrow MA = MC = AC = a$

מכאן $AB = a \Leftarrow \triangle ABC$ שווה צלעות $\Leftarrow AH = \frac{1}{2}a$

$\triangle ABH$: $BH = \sqrt{AB^2 - AH^2} = \frac{\sqrt{3}}{2}a$

$DB = 2BH = \sqrt{3}a$



$MH = \frac{\sqrt{3}}{2}a$

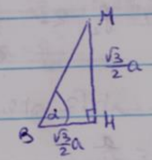
$\triangle MHB$: $MB = \sqrt{MH^2 + HB^2} = \sqrt{\frac{3a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{3}{2}}a$

כנ"ל עבור DM ב-$\triangle DMH$

$\triangle DMB$: $DB^2 = BM^2 + MB^2$ מתקיים

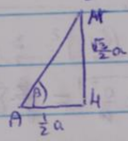
לכן הוא משולש ישר זווית.

(*) זווית בין MD ו- MB לכן 90°



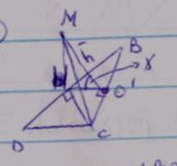
$\alpha = 45^\circ$

זווית בין MA ל- MC לכן 60°!



$\tan\beta = \frac{\frac{\sqrt{3}}{2}a}{\frac{1}{2}a} = \sqrt{3} \rightarrow \boxed{\beta = 60^\circ}$

(ב)

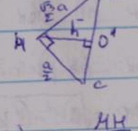


יש 3 נקודות הקרובות MO' ו- MO' נפגשים בנק' O'.

$S_{\triangle MBC} = \frac{BC \cdot MO'}{2} = \frac{a \cdot \sqrt{\frac{15}{16}}a}{2} = \frac{a^2\sqrt{15}}{8}$

$\uparrow$

$\triangle BOC$: $S_{\triangle BOC} = \frac{HB \cdot HC}{2} = \frac{\sqrt{3}a^2}{8} = \frac{BC \cdot \bar{h}}{2} \rightarrow \boxed{\bar{h} = \frac{\sqrt{3}a}{4}}$



$O'B = \sqrt{BH^2 - \bar{h}^2} = \sqrt{\frac{3a^2}{4} - \frac{3a^2}{16}} = \frac{3a}{4}$

$O'M = \sqrt{HM^2 + HO'^2} = \sqrt{\frac{3a^2}{4} + \frac{3a^2}{16}} = \sqrt{\frac{15}{16}}a$

$\sin\gamma = \frac{MH}{MO'} = \frac{\frac{\sqrt{3}}{2}a}{\sqrt{\frac{15}{16}}a} = \frac{2}{\sqrt{5}} \Rightarrow \cos\gamma = \sqrt{\frac{1}{5}} \rightarrow \boxed{\tan\gamma = 2}$

$V = \frac{1}{3} \cdot$ שטח הבסיס $\cdot MH = \frac{1}{3} \cdot a^2 \cdot \sin 60^\circ \cdot \frac{\sqrt{3}}{2}a = \frac{a^3}{4}$

$\measuredangle B$