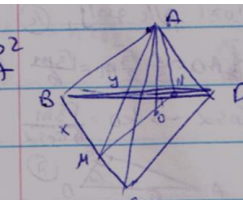
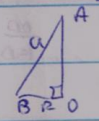
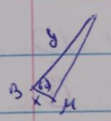
3.32
ד



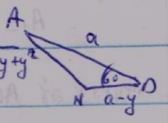
$R = \frac{\sqrt{3}}{3}a$ רדיוס המעגל המוסבר למשולש



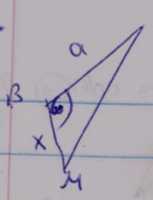
$AO = \sqrt{a^2 - R^2} = \sqrt{\frac{2}{3}}a$



$MN = \sqrt{x^2+y^2-2xy\cos 60} = \sqrt{x^2+y^2-xy}$



$AN = \sqrt{a^2+(a-y)^2-2a(a-y)\cos 60} = \sqrt{a^2-ay+y^2}$



$AM = \sqrt{a^2+x^2-2ax\cos 60} = \sqrt{a^2+x^2-ax}$

$\triangle AMO$: $MO = \sqrt{AM^2-AO^2} = \sqrt{\frac{1}{3}a^2+x^2-ax}$

$\triangle ANO$: $NO = \sqrt{AN^2-AO^2} = \sqrt{\frac{1}{3}a^2+y^2-ay}$

$MN = MO + NO$

$$\sqrt{x^2+y^2-xy} = \sqrt{\tfrac{1}{3}a^2+x^2-ax} + \sqrt{\tfrac{1}{3}a^2+y^2-ay} \quad /()^2$$

$$x^2+y^2-xy = \tfrac{2}{3}a^2+x^2+y^2-ax-ay+2\sqrt{\tfrac{1}{9}a^4-\tfrac{1}{3}a^3y-\tfrac{1}{3}a^3y+\tfrac{1}{3}a^2x^2-x^2ay+x^2y^2-\tfrac{1}{3}a^3x+\sqrt{+a^2xy-axy^2}}$$

$$ax+ay-xy-\tfrac{2}{3}a^2 = 2\sqrt{\qquad} \quad /()^2$$

$$a^2x^2+a^2y^2+x^2y^2+\tfrac{4}{9}a^4+2a^2xy-2ax^2y-\tfrac{1}{3}a^3x-2axy^2-\tfrac{4}{3}a^3y+\tfrac{4}{3}a^2xy =$$

$$= \tfrac{4}{9}a^4-\tfrac{4}{3}a^3y+\tfrac{4}{3}a^2y^2+\tfrac{4}{3}a^2x^2-4x^2ay+4x^2y^2-\tfrac{4}{3}a^2xy-4axy^2-\tfrac{4}{3}a^3x$$

$$0 = \tfrac{1}{3}a^2x^2+\tfrac{1}{3}a^2y^2+3x^2y^2+\tfrac{2}{3}a^2xy-2x^2ay-2axy^2 \quad /\cdot 3$$

$$0 = a^2(x^2+y^2)+9x^2y^2+2a^2xy-6x^2ay-6axy^2$$

$$0 = a^2x^2+2a^2xy+a^2y^2-6axy(x+y)+9x^2y^2$$

$$0 = (ax+ay)^2-6xay(x+y)+9x^2y^2$$

$$0 = [a(x+y)-3xy]^2$$

$$a(x+y) = 3xy \quad /:axy \rightarrow \quad \frac{1}{y}+\frac{1}{x} = \frac{3}{a}$$

ב) $\frac{5}{4a}+\frac{1}{y} = \frac{3}{a} \quad /\cdot 4ay$ (כפל ב)

$$5y+4a = 12y \rightarrow y = \frac{4a}{7}$$

$$MN = \sqrt{x^2+y^2-xy} = \sqrt{\frac{16a^2}{25}+\frac{16a^2}{49}-\frac{16a^2}{35}} = 4a\sqrt{\frac{1}{25}+\frac{1}{49}-\frac{1}{35}} = 4a\sqrt{\frac{49+25-35}{1225}} = \frac{4a}{35}\sqrt{39}$$

$$S_{\triangle NO} = \frac{AO\cdot MN}{2} = \frac{1}{2}\sqrt{\frac{2}{3}}a\cdot\frac{4a\sqrt{39}}{35} = \frac{2a^2\sqrt{26}}{35}$$