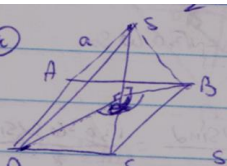
3.80
6

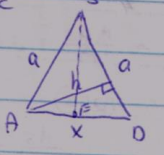
א)

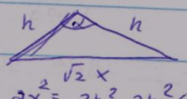


נסמן את אורך הגובה הפנימי ב- x

$$S_{ASD} = \frac{AD \cdot SE}{2} = \frac{x\sqrt{a^2 - \frac{x^2}{4}}}{2} = \frac{ha}{2} \rightarrow h = \frac{x}{a}\sqrt{a^2 - \frac{x^2}{4}}$$

$$\boxed{h = \frac{x}{2a}\sqrt{4a^2 - x^2}}$$





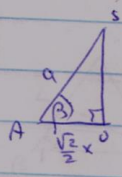
$$2x^2 = 2h^2 - 2h^2\cos\alpha$$

$$2x^2 = 2h^2(1-\cos\alpha) = \frac{2x^2}{4a^2}(4a^2 - x^2)(1-\cos\alpha)$$

$$4a^2 = (4a^2 - x^2)(1-\cos\alpha) = 4a^2 - x^2 - 4a^2\cos\alpha + x^2\cos\alpha$$

$$x^2 = \frac{4a^2\cos\alpha}{-1+\cos\alpha} \rightarrow x = 2a\sqrt{\frac{\cos\alpha}{-1+\cos\alpha}}$$

נסמן את גובה הפנימי ב- SO.



$$\cos\beta = \frac{AO}{AS} = \frac{\frac{\sqrt{2}}{2} \cdot 2a\sqrt{\frac{\cos\alpha}{-1+\cos\alpha}}}{a} = \sqrt{\frac{2\cos\alpha}{-1+\cos\alpha}} =$$

$$\sqrt{\frac{2\cos\alpha}{-2\sin^2\frac{\alpha}{2}}} = \sqrt{\frac{\cos\alpha}{-\sin^2\frac{\alpha}{2}}} = \sqrt{\frac{-\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}}} = \sqrt{1 - \cot^2\frac{\alpha}{2}}$$

ב) $\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-1+\cot^2\frac{\alpha}{2}} = \cot\frac{\alpha}{2} \rightarrow SO = a\sin\beta = a\cot\frac{\alpha}{2}$

$$V = \frac{1}{3} S_{ABCD} \cdot SO = \frac{1}{3} x^2 \cdot SO = \frac{1}{3} \cdot 4a^2 \cdot \frac{\cos\alpha}{\cos\alpha - 1} \cdot a\cot\frac{\alpha}{2} = \frac{1}{3} 4a^3 \cdot \frac{1}{2}\left(1 - \cot^2\frac{\alpha}{2}\right)\cot\frac{\alpha}{2}$$

$$\left[\frac{\cos\alpha}{\cos\alpha - 1} = \frac{\cos\alpha}{-2\sin^2\frac{\alpha}{2}} = \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}} = \frac{1}{2}\left(1 - \cot^2\frac{\alpha}{2}\right)\right]$$

$$= \frac{2a^3}{3}\cot\frac{\alpha}{2}\left(1 - \cot^2\frac{\alpha}{2}\right)$$