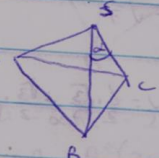
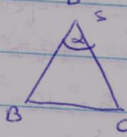


4.6  
6



المثلث المماس المكون له

$$S_{\text{المثلث}} = 2R^2 \sin \alpha \sin^2(90 - \frac{\alpha}{2}) = 2R^2 \sin \alpha \cos^2 \frac{\alpha}{2}$$



$$\frac{BC}{\sin \alpha} = 2R \rightarrow BC = 2R \sin \alpha$$

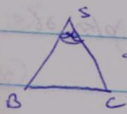
$$S_{ABC} = \frac{\sqrt{3}}{4} (BC)^2 = \frac{\sqrt{3}}{4} \cdot 4R^2 \sin^2 \alpha = \sqrt{3} R^2 \sin^2 \alpha$$

$$V = 3 \cdot S_{\text{المثلث}} \cdot h = 3 \cdot 2R^2 \sin \alpha \cos^2 \frac{\alpha}{2} \cdot \frac{\sqrt{3}}{3} R \sin \alpha = \sqrt{3} R^3 \sin^2 \alpha (\sqrt{3} \cos^2 \frac{\alpha}{2} + 1)$$

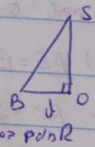
$$= \sqrt{3} R^3 \sin^2 \alpha (\frac{2\sqrt{3} \cos^2 \frac{\alpha}{2}}{2 \sin^2 \frac{\alpha}{2}} + 1) = \sqrt{3} R^3 \sin^2 \alpha (\sqrt{3} \cot^2 \frac{\alpha}{2} + 1)$$

(2)

$$R = \frac{\sqrt{3}}{3} BC = \frac{\sqrt{3}}{3} \cdot 2R \sin \alpha$$



$$\frac{BS}{\sin(90 - \frac{\alpha}{2})} = 2R \rightarrow BS = 2R \cos \frac{\alpha}{2}$$



منه نجد

$$SO = \sqrt{BS^2 - BO^2} = \sqrt{4R^2 \cos^2 \frac{\alpha}{2} - \frac{4}{3} R^2 \sin^2 \alpha} = 2R \sqrt{\cos^2 \frac{\alpha}{2} - \frac{1}{3} \sin^2 \alpha}$$

$$V = \frac{1}{3} S_{ABC} \cdot SO = \frac{1}{3} \cdot \sqrt{3} R^2 \sin^2 \alpha \cdot 2R \sqrt{\cos^2 \frac{\alpha}{2} - \frac{1}{3} \sin^2 \alpha} = \frac{2R^3 \sin^2 \alpha}{3} \sqrt{3 \cos^2 \frac{\alpha}{2} - \sin^2 \alpha}$$