

4.22
6

$r+h=a$

$\frac{h}{a-h} = \tan \alpha \rightarrow h = a \tan \alpha - h \tan \alpha$

$h = \frac{a \tan \alpha}{1 + \tan \alpha}$

(1)

∴ $\sqrt{3}$ \rightarrow $\frac{1}{3}$ \rightarrow $\frac{1}{3}$ \rightarrow $\frac{1}{3}$ \rightarrow $\frac{1}{3}$

$S_{ABC} = \frac{AC^2 \cdot \sqrt{3}}{4} = \frac{r(3AC)}{3} \rightarrow r = \frac{\sqrt{3}}{6} AC$ (2)

$AC = \frac{6}{\sqrt{3}} r = \frac{6}{\sqrt{3}} (a-h) = \frac{6}{\sqrt{3}} \left(a - \frac{a \tan \alpha}{1 + \tan \alpha} \right)$

$AC = \frac{6}{\sqrt{3}} \cdot \frac{a}{1 + \tan \alpha}$

$V = \frac{1}{3} \cdot S_{ABC} \cdot h = \frac{1}{3} \cdot \frac{\sqrt{3}}{4} \left(\frac{6}{\sqrt{3}} \cdot \frac{a}{1 + \tan \alpha} \right)^2 \cdot \frac{a \tan \alpha}{1 + \tan \alpha} = \frac{\sqrt{3} a^3 \tan \alpha}{(1 + \tan \alpha)^3}$