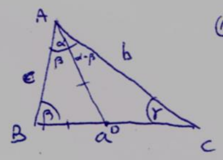
3.56
ב5



(א) $S_{ADC} = \dfrac{b^2 \sin\gamma \sin(\alpha-\beta)}{\sin 2\beta}$

$S_{ADC} = S_{ABC} - S_{ABD} = \frac{1}{2} ab\sin\beta - \frac{1}{2} c^2 \frac{\sin^2\beta}{2\sin\beta} =$ בדרך חלופית

$= \frac{1}{2} ab\sin\beta - \frac{1}{2} \frac{c^2 \sin^2\beta}{4\sin\beta\cos\beta} = \frac{1}{2} ab\sin\beta - \frac{1}{4} c^2 \frac{\sin\beta}{\cos\beta} = \frac{1}{4}(2ab\sin\beta - c^2\tan\beta)$

(ב) $S_{BDA} = \frac{1}{\cancel{2}} c^2 \cdot \frac{\sin^2\beta}{\sin 2\beta} = \frac{1}{4} c^2 \tan\beta$

(ג) $\underline{\triangle ABD}$: $\dfrac{AD}{\sin\beta} = \dfrac{c}{\sin 2\beta} \rightarrow AD = \dfrac{c}{2\cos\beta}$

$DC = BC - BD = a - \dfrac{c}{2\cos\beta}$

$\underline{\triangle ABC}$: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} \rightarrow a = \dfrac{b\sin\alpha}{\sin\beta}$

$\dfrac{S_{\triangle ADC}}{S_{\triangle ABD}} = \dfrac{\cancel{\frac{1}{2}AD} \cdot b \cdot \sin(\alpha-\beta)}{\cancel{\frac{1}{2}AD} \cdot c \cdot \sin\beta} = \dfrac{b\sin\alpha\cos\beta - b\cos\alpha\sin\beta}{c\sin\beta} =$

$= \dfrac{b\sin\alpha}{\sin\beta} \cdot \dfrac{\cos\beta}{c} - \dfrac{b\cos\alpha}{c} = \dfrac{a\cos\beta}{c} - \dfrac{b\cos\alpha}{c} = \dfrac{a\cos\beta - b\cos\alpha}{c}$