3.72
14

$a = 2R\sin\alpha$
$b = 2R\sin\beta$
$c = 2R\sin\gamma$

לפי משפט הסינוסים

$$p = \frac{a+b+c}{2} = R\sin\alpha + R\sin\beta + R\sin\gamma$$

נוכיח שהשוויון

$$\frac{\sin\frac{\alpha}{2}}{\cos\frac{\beta}{2}\cos\frac{\gamma}{2}} \stackrel{?}{=} \frac{2R\sin\alpha}{R\sin\alpha + R\sin\beta + R\sin\gamma}$$

$$\frac{\sin\frac{\alpha}{2}}{\cos\frac{\beta}{2}\cos\frac{\gamma}{2}} \stackrel{?}{=} \frac{2\sin\alpha}{\sin\alpha + \sin\beta + \sin\gamma}$$

$\gamma = 180 - \alpha - \beta$

$\sin\gamma = \sin(180 - \alpha - \beta)$

$\sin\gamma = \sin(\alpha + \beta)$

$$= \frac{2\sin\alpha}{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + \sin(\alpha+\beta)}$$

$$= \frac{2\sin\alpha}{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}}$$

$$= \frac{4\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}\right)}$$

$$= \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin\frac{\alpha+\beta}{2}\left(2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\right)}$$

$$= \frac{\sin\frac{\alpha}{2}}{\sin\frac{\alpha+\beta}{2}\cos\frac{\beta}{2}}$$

$$\cos\frac{\gamma}{2} = \cos\left(\frac{180-\alpha-\beta}{2}\right) =$$

$$= \cos\left(90 - \frac{\alpha+\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right)$$

$$= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\gamma}{2}\cos\frac{\beta}{2}}$$