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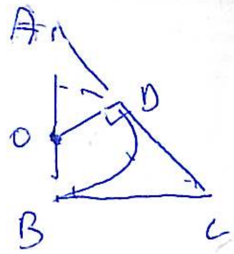
(683)



(ש.ש) $\triangle AOD \sim \triangle ABC$ (א)

$$\frac{OD}{BC} = \frac{AD}{AB}$$

נסמן את $AO = x$

$AD^2 = AO^2 - OD^2 = x^2 - r^2$

$AB = AO + OB = x + r$

$$\frac{r}{BC} = \frac{\sqrt{x^2-r^2}}{x+r}$$

$$BC = \frac{rx+r^2}{\sqrt{x^2-r^2}}, \quad f = S_{ABC} = \frac{BC \cdot AB}{2} = \frac{rx+r^2}{\sqrt{x^2-r^2}} \cdot \frac{x+r}{2} = \frac{r(x+r)^2}{2\sqrt{x^2-r^2}}$$

$$f' = \frac{2r(x+r)\cdot 2\sqrt{x^2-r^2} - \frac{r(x+r)^2 \cdot 2 \cdot 2x}{2\sqrt{x^2-r^2}}}{4(x^2-r^2)} = 0$$

נק' מינימום

x	r	$2r$	$3r$
y'	$-$		$+$
y	$\searrow$	min	$\nearrow$

$$4r(x+r)\sqrt{x^2-r^2} - \frac{4xr(x+r)^2}{2\sqrt{x^2-r^2}} = 0 \quad / 2\sqrt{x^2-r^2}$$

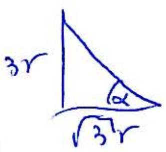
$$0 = 8r(x+r)(x^2-r^2) - 4xr(x+r)^2 = 4r(x+r)^2[2(x-r)-x] = 4r(x+r)^2(x-2r)$$

$\cancel{x = -r}$ $\quad x = 2r$

$BA = x + r = 2r + r = 3r$

(ב)

$$f = S_{ABC} = \frac{r(2r+r)^2}{2\sqrt{(2r)^2-r^2}} = \frac{r \cdot 9r^2}{2\sqrt{3r^2}} = \frac{9r^2}{2\sqrt{3}} = \frac{3\sqrt{3}}{2}r^2$$



$$\tan\alpha = \frac{3r}{\sqrt{3}r} = \sqrt{3} \rightarrow \alpha = 60 \quad (30, 60, 90)$$

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(נשענת על קוטר) $\angle C_1 = 90$

$BC = 2r\cos\alpha \Leftarrow \frac{BC}{AB} = \cos\angle B = \cos\alpha$

(מרובע חסום במעגל) $\angle D = 180 - \alpha$

במשולש ADC

$$\frac{\alpha}{2} = \frac{180-(180-\alpha)}{2} = \angle C_1 = \angle A_1 \Leftarrow$$

$AC = 2r\sin\alpha$, $AS = SC = \frac{1}{2}AC \Leftarrow$ (ש.ז.ז) $\triangle ADS \cong \triangle DCS$

$AS = \frac{1}{2}AC = r\sin\alpha$

$$\frac{AS}{AD} = \cos\frac{\alpha}{2} \rightarrow AD = \frac{r\sin\alpha}{\cos\frac{\alpha}{2}} = \frac{2r\cancel{\cos\frac{\alpha}{2}}\sin\frac{\alpha}{2}}{\cancel{\cos\frac{\alpha}{2}}} = 2r\sin\frac{\alpha}{2}$$

$$f = AD + DC + BC + AB = 2r\sin\frac{\alpha}{2} + 2r\sin\frac{\alpha}{2} + 2r\cos\alpha + 2r, \quad f' = \left(2r\cos\frac{\alpha}{2}\right)\cdot\frac{1}{2} + \left(2r\cos\frac{\alpha}{2}\right)\frac{1}{2} - 2r\sin\alpha$$

$$0 = 2r\cos\frac{\alpha}{2} - 2r\sin\alpha = 2r\cos\frac{\alpha}{2} - 4r\cos\frac{\alpha}{2}\sin\frac{\alpha}{2} = 2r\cos\frac{\alpha}{2}\left(1 - 2\sin\frac{\alpha}{2}\right)$$

$\frac{\alpha}{2} = \pm\frac{\pi}{2} + 2\pi k$

$\cancel{\alpha = \pm\pi + 4\pi k}$

לא מתאים

$\frac{\alpha}{2} = \frac{\pi}{6} + 2\pi k, \ \frac{5\pi}{6} + 2\pi k$

$\alpha = \frac{\pi}{3} + 4\pi k, \ \cancel{\frac{10\pi}{6} + 4\pi k}$

לא מתאים

בדקנו שבאמת $\alpha = 60$ הוא max.

$$f = 2r\sin30 + 2r\sin30 + 2r\cos60 + 2r = 5r$$