

68

נראה כי  $\sum_{k=1}^n \frac{1}{a_k - a_{k-1}}$  הוא קבוע

$$\begin{aligned} & \frac{1}{a_1 - a_2} \cdot \left( \frac{1}{a_2} - \frac{1}{a_1} \right) + \frac{1}{a_2 - a_3} \left( \frac{1}{a_3} - \frac{1}{a_2} \right) + \frac{1}{a_3 - a_4} \left( \frac{1}{a_4} - \frac{1}{a_3} \right) + \dots \\ & + \frac{1}{a_{n-1} - a_n} \left( \frac{1}{a_n} - \frac{1}{a_{n-1}} \right) = -\frac{1}{d} \left( \frac{1}{a_2} - \frac{1}{a_1} \right) + \frac{1}{d} \left( \frac{1}{a_3} - \frac{1}{a_2} \right) - \\ & \frac{1}{d} \left( \frac{1}{a_4} - \frac{1}{a_3} \right) - \dots - \frac{1}{d} \left( \frac{1}{a_n} - \frac{1}{a_{n-1}} \right) = \\ & -\frac{1}{d} \left( \frac{1}{a_2} - \frac{1}{a_1} + \frac{1}{a_3} - \frac{1}{a_2} + \frac{1}{a_4} - \frac{1}{a_3} + \dots + \frac{1}{a_n} - \frac{1}{a_{n-1}} \right) = \\ & -\frac{1}{d} \left( \frac{1}{a_n} - \frac{1}{a_1} \right) = -\frac{1}{d} \left( \frac{a_n - a_1}{a_n a_1} \right) = \\ & -\frac{1}{d} \left( \frac{a_n - a_1 - d(n-1)}{a_n a_1} \right) = \frac{1}{d} \cdot \frac{d(n-1)}{a_n a_1} = \frac{n-1}{a_n a_1} \end{aligned}$$